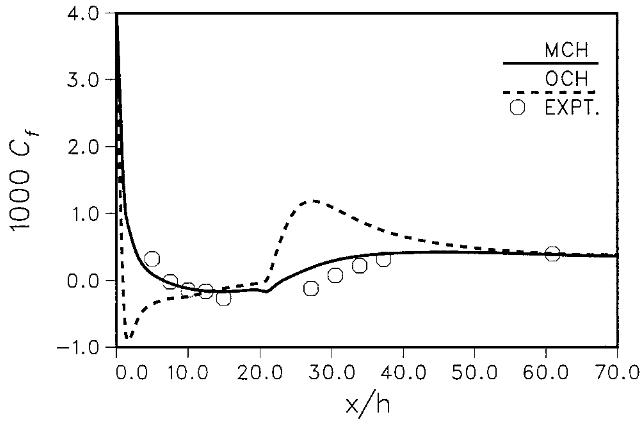

Fig. 3 Skin-friction coefficient of the diffuser flow along the deflected bottom wall.

predicted skin-friction coefficients C_f. The superior performance of the MCH model, in strong contrast to that of the OCH model, is once more ascertained. Apparently, the ambiguous prediction regarding the OCH model is attributable to shortcomings in the y^+ dependence viscous damping functions employed.

Conclusion

The potential importance of the cross diffusion together with the viscous damping functions is conspicuous. The modification introduced with the Chien model is profoundly convenient because it circumvents the defect entangled with the model to a greater extent. The MCH model accounts for the near-wall and low-Reynolds-number effects emanating from the physical requirements.

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Vibration of Thermally Stressed Composite Plates with and Without Cutouts

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I. Introduction

STRUCTURES made of composite materials often operate at elevated temperatures. At these temperatures thermomechanical stressing may occur. Composite structures that are thermally stressed can resonate at various frequencies. It is therefore of interest to examine the vibration behavior of structures made of composite materials that are thermally stressed. Of particular interest is the effect of initial thermal stressing to the subsequent oscillating behavior of composite panels. The present study considers two laminated composite plates; an eight-layer $(0/90/0/90)_s$ cross-ply laminate and an eight-layer quasi-isotropic $(45/-45/0/90)_s$ composite plate. Both plates are thermally stressed via the application of various temperatures. Following thermal stressing, an eigenvalue problem is considered, and the first natural frequency of the structure is extracted.

Researchers have started to study the vibration behavior of composite structures at elevated temperatures.[1–5] The frequency-temperature curves are nonlinear in nature and are affected by the particular lamination. The present study considers the effect of the temperature on the fundamental natural frequency of laminated composite plates with and without cutouts. The effect of thermal stressing on the fundamental frequency of composite plates with holes is compared with the corresponding effect of laminates without cutouts.

II. Computational Experiments

Figure 1 shows a laminated composite plate along with all geometrical and material properties. The left edge of the plate cannot move in the three directions, whereas in all other edges the vertical displacement is prohibited. Uniform temperature increase is applied on the top and bottom boundaries. A small central cutout is considered as depicted in the figure. Two eight-layer laminations are considered, namely cross-ply $(0/90/0/90)_s$ and quasi-isotropic $(45/-45/0/90)_s$. The application of temperature introduces the geometrically nonlinear problem

$$K_T r = J \tag{1}$$

where K_T is the tangent stiffness and J is the initial load caused by temperature. The temperature is applied incrementally, and following convergence and the full application of the incremental temperature the following eigenvalue problem is solved:

$$K_T x = \lambda M x \tag{2}$$

where M is the global mass matrix, λ the eigenvalue (natural frequency), and x the eigenvector.

The composite plate is discretized with a set of triangular elements based on the natural-mode finite element method.[6] In brief, the triangular finite element is assigned a set of rigid-body and straining modes. The latter are equal to the global nodal degrees of freedom minus the number of rigid-body modes.

Figure 2 shows the temperature-frequency curves for an eight-layer $(0/90/0/90)_s$ cross-ply laminate with and without cutouts. We observe that from $T = 0$ to 120°C the presence of the cutout results in a decrease of the natural frequency of the plate. At subsequent frequencies however, the natural frequency of the plate with the cutout

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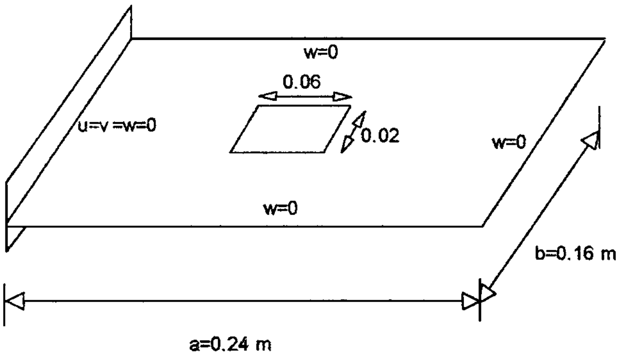

Fig. 1 Laminated composite plate; geometrical and material data: $E_1 = 150$ GPa; $E_2 = E_3 = 10$ GPa; $G_{12} = G_{13} = 6$ GPa; $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$; $a_{t1} = 2.5 \times 10^{-8}\,°\mathrm{C}^{-1}$; $a_{t2} = 30 \times 10^{-6}\,°\mathrm{C}^{-1}$; and $h_L = 3.125 \times 10^{-4}$ m.

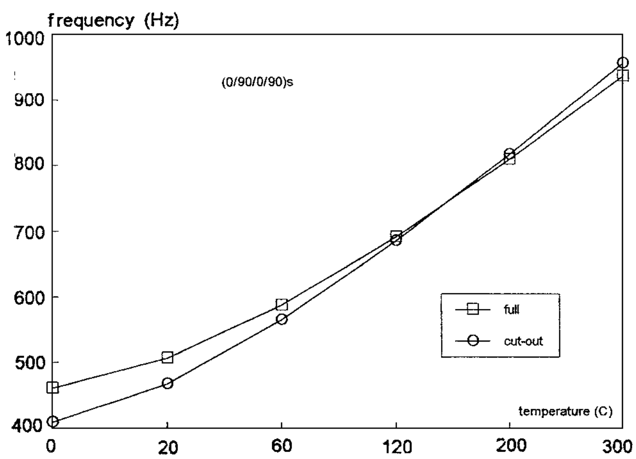

Fig. 2 Temperature-frequency curves for the $(0/90/0/90)_s$ laminate with and without cutout.

surpasses the fundamental frequency of the plate without the central hole. This can be attributed to the significant thermal stressing that occurs at higher temperatures in the plate with the cutout. In other words, the cutout allows the plate to expand and get stressed more than the plate without the cutout. Figure 3 shows the temperature-frequency curves for the quasi-isotropic $(45/-45/0/90)_s$ laminate with and without the central cutout. The curves follow a similar trend; as for the $(0/90/0/90)_s$ laminate, however, the effect of the cutout at the lower temperatures is not so pronounced as it was for the cross-ply composite plate. At temperatures higher than $T = 120°\mathrm{C}$, the natural frequencies of the plate with the cutout are higher than those of the full plate. The same reasoning as before applies.

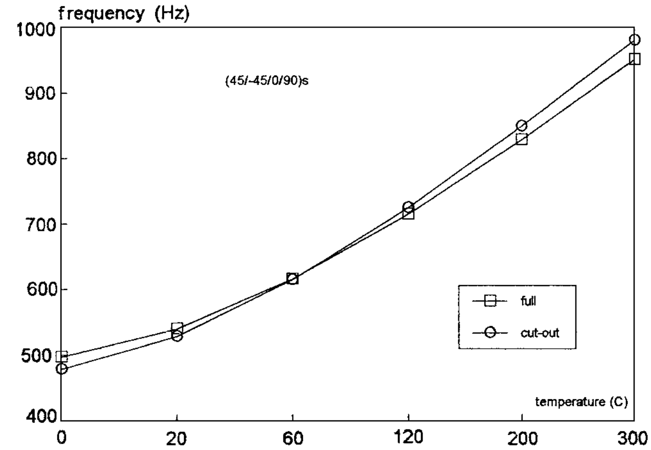

Fig. 3 Temperature-frequency curves for the $(45/-45/0/90)_s$ laminate with and without cutout.

III. Conclusions

The frequency response of two thermally stressed composite laminates is examined. Both laminates include a small central cutout. Initially, the laminates are considered thermally stressed via the application of various incremental temperatures. At full application of temperatures, their fundamental natural frequencies are extracted. For temperatures up to $T = 120°\mathrm{C}$, the presence of the cutout results in a drop of the natural frequency. This frequency reduction is more pronounced for the cross-ply laminate. At higher temperatures the frequencies of the plates with the cutout surpass those of the full plate—an indication of significant thermal stressing.

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